Journal of Fluids and Structures (1998) **12,** 549–559
Article No. fl980158

HYDRODYNAMIC FORCE ON A RIGID BODY DURING IMPACT WITH LIQUID

G. X. Wu

Department of Mechanical Engineering, University College London, Torrington Place London, WC1E 7JE, U.K.

(Accepted 22 October 1997 and in revised form 17 March 1998)

This paper has resolved the difference between two well-known equations in marine hydrodynamics. It shows that when a rigid body enters a liquid, the forces obtained by integrating the pressure over the body surface and by an energy argument are in fact identical.

1. INTRODUCTION

Based on velocity potential theory, it is well known that when a rigid body moves in an unbounded fluid domain, the hydrodynamic force on the body is equal to its added mass multiplied by its acceleration. When a body enters into a liquid (fluid/structure impact), the situation is somewhat different. The case may seem simple enough, but different equations for calculating the force have emerged. A well-known example is the difference between the results of Faltinsen (1990; p. 286) and Miloh (1981). They both adopted the same mathematical model in which the free surface is approximated by a flat plane and the potential on that plane is zero during impact, but Faltinsen obtained the force by integrating the pressure over the body surface, while Miloh obtained the force based on an energy argument. It was found that when the speed of the body is constant, their results differ by a factor of two. The problem has been further highlighted by Molin, Cointe & Fontaine (1996). They considered a case of a circular cylinder and argued that the contradiction could be resolved by including a jet developed during the impact. This however, seems, not to be the full solution to the problem. There are still a few questions unanswered: (a) whether including a jet will resolve the contradiction for other geometries; (b) more importantly, neither Faltinsen nor Miloh has included the jet in their analyses, and one cannot add the contribution from the jet to one equation but not to the other; and (c) how to resolve the difference for a fully submerged body where no jet is present.

In this work, we shall show that the force obtained from the pressure integration over the body surface is the same as that obtained from the energy argument. It has to be stressed that the present analysis does not attempt to offer any new mathematical model for the fluid/structure impact problem. The purpose here is to show that, based on the mathematical model used by Faltinsen (1990) and Miloh (1981), there should be one equation for the impact force. Whether their mathematical model reflects the physics of the problem is not the main concern here. Having said that, this model can be traced back to von Karman (1929) and Lamb (1932). It has been used ever since for various impact problems. More recent applications include that by Cooker & Peregrine (1995) for wave impact on a wall.

0889–9746/98/050549 + 11 $30.00

Although this model is based on some drastic simplifications from a hydrodynamic point of view, it does give some useful results for the loading on the structure in many cases [see, for example, Faltinsen (1990)], which is the main concern of this paper. A more accurate model which includes the deformation of the free surface has been used by Zhao & Faltinsen (1992) and Korobkin (1997).

2. COVERNING EQUATION

We define a Cartesian system O-xyz, with the origin being on the undisturbed free surface and z pointing upwards. We consider the problem of a body entering water with vertical speed W (negative W means that the body moves downwards). The mathematical model is based on the theory for an incompressible and irrotational flow, using a velocity potential. The free surface is approximated by $z = 0$ and the potential on this flat surface is assumed to be zero during the impact. The velocity potential then satisfies the following equation:

$$\nabla^2 \phi = 0 \tag{1}$$

in the fluid domain R bounded by $z = 0$, the body surface and the bottom;

$$\phi = 0 \tag{2}$$

on the free surface S_F which is approximated by $z = 0$;

$$\frac{\partial \phi}{\partial n} = W n_z \tag{3}$$

on the body surface S_0, where $\mathbf{n}$ is the inward normal of the body surface and n_z is its component in z direction;

$$\frac{\partial \phi}{\partial n} = 0 \tag{4}$$

at the bottom of the fluid S_B.

It should be noted that equation (2) automatically leads to $\phi_t = 0$ when $z = 0$, which will be used in the following derivation. There are two points which should be emphasized here: (i) since we have imposed the condition in equation (2), we can no longer use $\partial\phi/\partial z = 0$ on $z = 0$ and (ii) since the free surface is approximated by a flat surface $z = 0$, the normal velocity of the surface deformation, U_n, is therefore zero. It is important to understand here that the meanings of $\partial\phi/\partial z$ and U_n are different and one cannot automatically assume $\partial\phi/\partial z = U_n$ on $z = 0$. It is also important to understand that on $z = 0$ the pressure $p = -\rho(\phi_t + \frac{1}{2}\nabla\phi\ \nabla\phi + gz) = -\frac{1}{2}\rho\phi_z^2$ is not zero, where ρ is the density of the fluid. Strictly speaking, therefore, $z = 0$ is not a free surface but an equipotential surface in the hydrodynamic sense. This surface together with the condition on it is used as an approximation to the real free surface during the impact.

3. FORCE BY INTEGRATING PRESSURE

The force on the body can be found by integrating the pressure over its wetted surface (below $z = 0$ in this model):

$$F = -\rho \int_{S_0} (\phi_t + \tfrac{1}{2}\nabla\phi\ \nabla\phi)\, n_z\, \mathrm{d}S, \tag{5}$$

in which the hydrostatic term has been ignored. The derivative with respect to time can be rewritten using the following transport theorem (Newman 1977; p. 57):

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{R(t)} f\mathrm{d}R = \int_{R(t)} \frac{\partial f}{\partial t}\,\mathrm{d}R + \int_{S(t)} fU_n\,\mathrm{d}S,$$

where $S(t)$ is the boundary of $R(t)$ and U_n is the normal velocity of the surface $S(t)$. It should be noted that $S(t)$ in this equation does not have to be a material surface. Also U_n depends solely on $S(t)$ and is not always equal to the normal velocity of the fluid particle on $S(t)$. Since $\phi = 0$ on S_F, it follows that

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0} \phi n_z\,\mathrm{d}S &= \frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0+S_F} \phi n_z \mathrm{d}S\\ &= \frac{\mathrm{d}}{\mathrm{d}t}\int_{R} \phi_z\,\mathrm{d}R - \frac{\mathrm{d}}{\mathrm{d}t}\int_{S_C} \phi n_z\,\mathrm{d}S\\ &= \int_{S_0+S_F+S_C} \phi_t n_z\,\mathrm{d}S + \int_{S_0+S_F+S_C} \phi_z U_n\,\mathrm{d}S - \frac{\mathrm{d}}{\mathrm{d}t}\int_{S_C} \phi n_z\,\mathrm{d}S,\end{aligned}$$

where S_C is a control surface and it can be dropped from the above equation for the reason described by Newman (1977; p. 133). Following the above discussion about U_n and that after equation (4), we have $U_n = 0$ on S_F and $U_n = \partial\phi/\partial n$ on S_0 and thus

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0} \phi n_z \mathrm{d}S = \int_{S_0} \phi_t n_z\,\mathrm{d}S + \int_{S_0} \phi_z \frac{\partial\phi}{\partial n}\,\mathrm{d}S,$$

in which $\phi_t = 0$ on S_F has been used. Substituting this into equation (5), we have

$$F = -\rho\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0} \phi n_z\,\mathrm{d}S + \rho\int_{S_0}\left(\frac{\partial\phi}{\partial n}\phi_z - \tfrac{1}{2}\nabla\phi\,\nabla\phi n_z\right)\mathrm{d}S. \tag{6}$$

Equation (6) can also be derived by an alternative procedure. We may use

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0(t)} \phi(x, y, z\ t) n_z\,\mathrm{d}S\\ &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\int_{S_0(t+\Delta t)} \phi(x, y, z+W\Delta t, t+\Delta t) n_t\,\mathrm{d}S - \int_{S_0(t)} \phi(x, y, z, t) n_z\,\mathrm{d}S\right),\end{aligned}$$

where the point (x, y, z) is fixed on the body. This gives

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0(t)} \phi(x, y, z\ t) n_z\,\mathrm{d}S\\ &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\int_{S_0(t+\Delta t)-S_0(t)} \phi(x, y, z+W\Delta t, t+\Delta t) n_z\,\mathrm{d}S\right.\\ &\quad\left.+\int_{S_0(t)} [\phi(x, y, z+W\Delta t, t+\Delta t) - \phi(x, y, z, t)]\, n_z\,\mathrm{d}S\right).\end{aligned}$$

If we use $S_0(t+\Delta t) - S_0(t) = -C_0(t)W\Delta t/\sqrt{n_x^2+n_y^2}$ where $C_0(t)$ is the waterline of the body, on which the components of the normal are taken, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0(t)} \phi(x, y, z\ t) n_z\, \mathrm{d}S$$
$$= -W\int_{C_0(t)} \phi(x, y, 0, t) n_z/\sqrt{n_x^2+n_y^2}\, \mathrm{d}C + \int_{S_0(t)} \left[W\frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial t}\right] n_z\, \mathrm{d}S.$$

Since $\phi = 0$ when $z = 0$, we have

$$\int_{S_0(t)} \frac{\partial\phi}{\partial t} n_z\, \mathrm{d}S = \frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0(t)} \phi n_z\, \mathrm{d}S - \int_{S_0(t)} \frac{\partial\phi}{\partial z} W n_z\, \mathrm{d}S$$
$$= \frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0(t)} \phi n_z\, \mathrm{d}S - \int_{S_0(t)} \frac{\partial\phi}{\partial z}\frac{\partial\phi}{\partial n}\, \mathrm{d}S.$$

Substituting this into equation (5) we obtain equation (6).

Equation (6) can also be writen as

$$F = -\frac{\mathrm{d}}{\mathrm{d}t}(M_a W) + \rho\int_{S_0}\left(\frac{\partial\phi}{\partial n}\phi_z - \tfrac{1}{2}\nabla\phi\,\nabla\phi n_z\right)\mathrm{d}S, \tag{7}$$

where M_a is the added mass defined as

$$M_a = \rho\int_{S_0} \psi_{t_z}\, \mathrm{d}S \tag{8}$$

and $\psi = \phi/W$.

4. ENERGY ARGUMENT

The kinetic energy in the fluid is

$$E = \tfrac{1}{2}\rho\int_R \nabla\phi\,\nabla\phi\, \mathrm{d}R = \tfrac{1}{2}\rho\int_S \phi\frac{\partial\phi}{\partial n}\, \mathrm{d}S, \tag{9}$$

where $S = S_0 + S_F + S_B$. Using the boundary conditions on S_F and on S_B in equations (2) and (4), we have

$$E = \tfrac{1}{2}\rho\int_{S_0}\phi\frac{\partial\phi}{\partial n}\, \mathrm{d}S = \tfrac{1}{2}\rho W^2\int_{S_0}\psi n_z\, \mathrm{d}S = \tfrac{1}{2} M_a W^2. \tag{10}$$

We now apply the transport theorem to equation (9):

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \tfrac{1}{2}\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_R \nabla\phi\,\nabla\phi\, \mathrm{d}R\right) = \rho\int_R \nabla\phi_t\,\nabla\phi\, \mathrm{d}R + \tfrac{1}{2}\rho\int_S \nabla\phi\,\nabla\phi U_n \mathrm{d}S. \tag{11}$$

As discussed after equation (4), $U_n = 0$ on S_F (but $\partial\phi/\partial z \neq 0$). At the bottom, $U_n = \partial\phi/\partial n = 0$, and on the body surface $U_n = \partial\phi/\partial n = W n_z$. Thus, equation (11) becomes

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \rho\int_S \phi_t\frac{\partial\phi}{\partial n}\, \mathrm{d}S + \tfrac{1}{2}\rho W\int_{S_0}\nabla\phi\,\nabla\phi\, n_z\, \mathrm{d}S.$$

Using the boundary conditions in equations (2), (3) and (4), we have

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \rho W \int_{S_0} (\phi_t + \tfrac{1}{2}\nabla\phi\ \nabla\phi)\, n_z\, \mathrm{d}S = -FW \tag{12}$$

Substituting equation (10) into equation (12), we obtain

$$F = -\tfrac{1}{2}\frac{\mathrm{d}M_a}{\mathrm{d}t} W - M_a \frac{\mathrm{d}W}{\mathrm{d}t}. \tag{13}$$

This equation may appear to be quite different from (7). As we shall show below, they are in fact identical.

5. AN IDENTITY

We shall prove the following identity:

$$\begin{aligned}\frac{\mathrm{d}M_a}{\mathrm{d}t} &= \frac{2\rho}{W}\int_{S_0}\left(\frac{\partial\phi}{\partial n}\phi_z - \tfrac{1}{2}\nabla\phi\ \nabla\phi\, n_z\right)\mathrm{d}S \\ &= 2\rho W \int_{S_0}\left(\frac{\partial\psi}{\partial n}\psi_z - \tfrac{1}{2}\nabla\psi\ \nabla\psi\, n_z\right)\mathrm{d}S.\end{aligned} \tag{14}$$

In fact, based on the definition of M_a and following the procedure leading to equation (6), we have

$$\frac{\mathrm{d}M_a}{\mathrm{d}t} = \rho\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0}\psi n_z\,\mathrm{d}S = \rho\int_{S_0}\psi_t n_z \mathrm{d}S + \rho W\int_{S_0}\psi_z n_z\,\mathrm{d}S. \tag{15}$$

The first term on the right-hand side can be written as

$$\begin{aligned}\int_{S_0}\psi_t n_z\,\mathrm{d}S &= \int_{S_0}\left(\psi_t\frac{\partial\psi}{\partial n} - \frac{\partial\psi_t}{\partial n}\psi\right)\mathrm{d}S + \int_{S_0}\frac{\partial\psi_t}{\partial n}\psi\,\mathrm{d}S \\ &= -\int_{S_F+S_B}\left(\psi_t\frac{\partial\psi}{\partial n} - \frac{\partial\psi_t}{\partial n}\psi\right)\mathrm{d}S + \int_{S_0}\frac{\partial\psi_t}{\partial n}\psi\,\mathrm{d}S \\ &= \int_{S_0}\frac{\partial\psi_t}{\partial n}\psi\,\mathrm{d}S,\end{aligned}$$

after using conditions on S_F and S_B. From Wu & Eatock Taylor (1996) or from the Appendix [equation (A7)], we have

$$\frac{\partial\phi_t}{\partial n} = \frac{\mathrm{d}W}{\mathrm{d}t}n_z - W\frac{\partial\phi_z}{\partial n}.$$

This leads to

$$\begin{aligned}\frac{\partial\psi_t}{\partial n} &= \frac{\partial}{\partial n}\left[\frac{\partial}{\partial t}\left(\frac{\phi}{W}\right)\right] = \frac{\partial}{\partial n}\left[-\frac{\phi}{W^2}\frac{\mathrm{d}W}{\mathrm{d}t} + \frac{1}{W}\frac{\partial\phi}{\partial t}\right] \\ &= -\frac{1}{W}\frac{\mathrm{d}W}{\mathrm{d}t}n_z + \frac{1}{W}\frac{\partial\phi_t}{\partial n} = -W\frac{\partial\psi_z}{\partial n}.\end{aligned}$$

Thus,

$$\int_{S_0} \psi_t \, n_z \, \mathrm{d}S = -W \int_{S_0} \frac{\partial \psi_z}{\partial n} \psi \, \mathrm{d}S.$$

Furthermore, since (Wu & Eatock Taylor 1996)

$$\int_{S_0} \frac{\partial \psi_z}{\partial n} \psi \, \mathrm{d}S = \int_{S_0} \left(\nabla\psi \; \nabla\psi n_z - \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial n} \right) \mathrm{d}S,$$

we have

$$\int_{S_0} \psi_t \, n_z \, \mathrm{d}S = W \int_{S_0} \left(\frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial n} - \nabla\psi \; \nabla\psi \, n_z \right) \mathrm{d}S. \tag{16}$$

Substituting this equation into equation (15) and noting that $\partial\psi/\partial n = n_z$, we can immediately obtain (14). Using this identity and comparing equations (7) and (13), we can see that these two equations are indeed identical.

Based on the method of the equivalent flat plate, Faltinsen (1990; p. 286) showed that

$$F = -\frac{\mathrm{d}M_a}{\mathrm{d}t} W \tag{17}$$

when $W =$ constant. This differs from equation (13) by a factor of two. In the derivation by Faltinsen, however, it was thought that the contribution from the ϕ_t term is far larger than that from the $\nabla\phi \; \nabla\phi$ term and the latter can be neglected. This might be true for the pressure, but equation (16) clearly shows that such an assumption is not correct for the force, or the integrated result (see also Section 6.2). Equation (17) therefore is an approximation and differs from the correct solution by a factor of two.

Equation (13) may seem to contradict the well-known equation for rigid-body motion,

$$F_e = \frac{\mathrm{d}}{\mathrm{d}t}(MW),$$

where F_e is the external force on the body and M is the body mass. However, equation (13) is, in fact, the force applied by the body only to the fluid. There will be a force from other boundaries of the fluid. For this reason, we cannot automatically expect that the momentum change of the fluid is equal to the force applied by the body.

6. DISCUSSION

6.1. Fully Nonlinear Case

It is important to notice that the foregoing discussion has been based on the assumption that the free surface is approximated by a flat surface on which the potential is zero during the impact. When the following fully nonlinear free surface boundary conditions are used:

$$\phi_t + \tfrac{1}{2} \nabla\phi \; \nabla\phi + g\zeta = 0, \tag{18}$$

$$\phi_z = \zeta_t + \phi_x \zeta_x + \phi_y \zeta_y, \tag{19}$$

where $\zeta(x, y, t)$ is the free surface elevation and g is the acceleration due to gravitation, the force should be obtained from the equation derived by Wu & Eatock Taylor (1996),

$$F = -M_a \frac{dW}{dt} - W \int_{S_0} \nabla\psi \, \nabla(\phi - Wz) n_z \, dS - \int_{S_0 + S_F} \left(\tfrac{1}{2} \nabla\phi \, \nabla\phi + gz \right) \frac{\partial \psi}{\partial n} \, dS, \quad (20)$$

in which ψ satisfies the Laplace equation and the following free-surface and body-surface boundary conditions hold:

$$\psi(x, y, z, t) = 0 \quad \text{on } z = \zeta(x, y, t), \quad (21)$$

and

$$\frac{\partial \psi}{\partial n} = n_z \quad \text{for } (x, y, z) \in S_0 . \quad (22)$$

It should also be pointed out that the hydrostatic term is included in equation (20).

For this case

$$\begin{aligned} \frac{dM_a}{dt} &= \rho \frac{d}{dt} \int_{S_0} \psi n_z \, dS = \rho \frac{d}{dt} \int_{S_0 + S_F} \psi n_z \, dS \\ &= \rho \int_{S_0 + S_F} \psi_t n_z \, dS + \rho \int_{S_0 + S_F} \psi_z U_n \, dS, \end{aligned} \quad (23)$$

where $U_n = \partial\phi/\partial n$ even on S_F, because the exact free surface boundary condition is applied. For the same reason $\psi_t \neq 0$ on S_F. In fact, because $\psi(x, y, \zeta, t) = 0$, we have

$$\psi_t + \psi_z \zeta_t = 0 \quad \text{on } z = \zeta(x, y, t). \quad (24)$$

Using $U_n = \zeta_t n_z$, equation (23) becomes

$$\frac{dM_a}{dt} = \rho \int_{S_0} \psi_t n_z \, dS + \rho \int_{S_0} \psi_z U_n \, dS. \quad (25)$$

Now, following the analysis in Section 5, we have

$$\begin{aligned} \int_{S_0} \psi_t n_z \, dS &= \int_{S_0} \left(\psi_t \frac{\partial \psi}{\partial n} - \frac{\partial \psi_t}{\partial n} \psi \right) dS + \int_{S_0} \frac{\partial \psi_t}{\partial n} \psi \, dS \\ &= -\int_{S_F + S_B} \left(\psi_t \frac{\partial \psi}{\partial n} - \frac{\partial \psi_t}{\partial n} \psi \right) dS + \int_{S_0} \frac{\partial \psi_t}{\partial n} \psi \, dS \\ &= -\int_{S_F} \psi_t \frac{\partial \psi}{\partial n} \, dS + \int_{S_0} \frac{\partial \psi_t}{\partial n} \psi \, dS \\ &= -\int_{S_F} \psi_t \frac{\partial \psi}{\partial n} \, dS - W \int_{S_0} \frac{\partial \psi_t}{\partial n} \psi \, dS \\ &= -\int_{S_F} \psi_t \frac{\partial \psi}{\partial n} \, dS + W \int_{S_0} \left(\frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial n} - \nabla\psi \, \nabla\psi \, n_z \right) dS. \end{aligned} \quad (26)$$

Substituting this into equation (25), we have

$$\frac{dM_a}{dt} = -\rho \int_{S_F} \psi_t \frac{\partial \psi}{\partial n} \, dS + 2\rho W \int_{S_0} \left(\frac{\partial \psi}{\partial n} \psi_z - \tfrac{1}{2} \nabla\psi \, \nabla\psi \, n_z \right) dS. \quad (27)$$

Comparing this with equation (14), we find that there is an extra term here involving the integration over the free surface.

6.2. Impact at $t = 0$

The fluid/structure impact can be divided into two stages: (i) from $t = 0^-$ to $t = 0^+$, and (ii) $t > 0^+$. What has been discussed here corresponds to the second stage. When $0^- \leq t \leq 0^+$, the force impulse can be found as (Batchelor 1967; p. 471)

$$\int_{0^-}^{0^+} F \, \mathrm{d}t = -M_a W|_{t=0}. \tag{28}$$

This equation seems to suggest that the equation for force should be

$$F = -\frac{\mathrm{d}(M_a W)}{\mathrm{d}t}. \tag{29}$$

In fact, within $0^- \leq t \leq 0^+$ one may assume that the second term in equation (7) is much smaller than the first one and its contribution may be ignored, as done by Faltinsen (1990). The above suggestion appears to be valid, although equation (29) itself may have no practical relevance. When $t > 0^+$, equation (14) clearly shows that such a suggestion is no longer valid.

6.3. Body Moving near A Rigid Wall

when a body is moving near a rigid wall, equation (13) is also valid, as shown by Lamb (1932; Art. 137). The difference is in the definitions of the added mass. Although the added masses in both cases are given by equation (8), the meanings of ψ are different. Here $\psi = 0$ on $z = 0$, while for the case of a body moving near a rigid wall, $\partial\psi/\partial z = 0$ on $z = 0$.

6.4. Lagrange Equation of Motion for a Body near a Flat Equipotential Surface

Lamb (1932; p. 190) obtained the force on a body by using the Lagrange equation of motion. We shall verify that this equation is valid in the special case considered in this paper: a body near a flat equipotential surface. The kinetic energy T can be obtained from

$$T = \tfrac{1}{2}\rho \int_R \nabla\phi \, \nabla\phi \, \mathrm{d}R = \tfrac{1}{2}\rho \int_{S_0} \phi \frac{\partial\phi}{\partial n} \, \mathrm{d}S, \tag{30}$$

where the boundary conditions on the free surface and on the bottom have been used. Using the definition in equation (8) and the boundary condition on the body surface, we have

$$T = \tfrac{1}{2} M_a W^2. \tag{31}$$

The Lagrange equation of motion gives (Lamb 1932; p. 190)

$$F = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial W}\right) + \frac{\partial T}{\partial X_3}, \tag{32}$$

where

$$X_3 = \int W \; \mathrm{d}t. \tag{33}$$

Substituting equation (33) into (32), we have

$$\begin{aligned} F &= -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial W}\right) + \frac{\partial T}{\partial X_3} \\ &= -\frac{\mathrm{d}}{dt}(M_a W) + \tfrac{1}{2} W^2 \frac{\mathrm{d}M_a}{\mathrm{d}X_3} \\ &= -\frac{\mathrm{d}}{\mathrm{d}t}(M_a W) + \tfrac{1}{2} W^2 \frac{\mathrm{d}M_a}{\mathrm{d}t} \frac{1}{\mathrm{d}X_3/\mathrm{d}t} \\ &= -M_a \frac{\mathrm{d}W}{\mathrm{d}t} - \tfrac{1}{2} W \frac{\mathrm{d}M_a}{\mathrm{d}t}, \end{aligned}$$

which is identical to equation (13).

7. CONCLUSIONS

This work has shown that two different equations for calculating the force on a body entering water are in fact the same. One by-product of the paper is equation (14). In numerical calculation, the left-hand side is not easy to deal with, when all physical parameters change sharply with time during impact. The right-hand side, on the other hand, is much easier to calculate.

ACKNOWLEDGEMENT

The author has been greatly benefited from some stimulating discussions with Prof. T. Miloh and Prof. R.C.T. Rainey. The work also forms part of an EPSRC project (GR/K61135).

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APPENDIX

We consider here the problem that the body moves with translational velocity **U** and rotational velocity Ω. The body surface condition for ϕ can then be written as

$$\frac{\partial}{\partial n}[\phi(x, y, z, t)] = (\mathbf{U} + \mathbf{\Omega} \times \mathbf{r}) \cdot \mathbf{n}, \tag{A1}$$

where **r** is the position vector from the point where **U** is measured.

In taking the derivative with respect to t in equation (A1), it is important to notice that $(x, y, z) \in S_0(t)$ and therefore they are a function of time when the body is in motion. We write

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial n}\right) &= \frac{d}{\mathrm{d}t}(\nabla\phi \cdot \mathbf{n}) \\ &= \frac{\mathrm{d}}{\mathrm{d}t}(\nabla\phi)\cdot\mathbf{n} + \nabla\phi\cdot\frac{\mathrm{d}\mathbf{n}}{\mathrm{d}t}\end{aligned} \tag{A2}$$

It should be pointed out that the meaning of $\mathrm{d}/\mathrm{d}t$ in the above equation is different from that used in the Lagrangian description of fluid motion. The former follows a fixed point on the body while the latter follows a given fluid particle. Thus, it is easy to establish that

$$\frac{\mathrm{d}\mathbf{n}}{\mathrm{d}t} = \mathbf{\Omega} \times \mathbf{n} \tag{A3}$$

and

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(\nabla\phi) &= \nabla\phi_t + \left(\frac{\mathrm{d}\mathbf{r}_a}{\mathrm{d}t}\cdot\nabla\right)\nabla\phi \\ &= \nabla\phi_t + [(\mathbf{U} + \mathbf{\Omega}\times\mathbf{r})\cdot\nabla]\,\nabla\phi,\end{aligned} \tag{A4}$$

where $\mathbf{r}_0$ is a position vector from O and therefore $\mathrm{d}\mathbf{r}_0/\mathrm{d}t = \mathbf{U} + \mathbf{\Omega}\times\mathbf{r}$. Substituting equations (A3) and (A4) into equation (A2), we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial n}\right) &= \nabla\phi_t\cdot\mathbf{n} + \{[(\mathbf{U} + \mathbf{\Omega}\times\mathbf{r})\cdot\nabla]\,\nabla\phi\}\cdot\mathbf{n} + \nabla\phi\cdot(\mathbf{\Omega}\times\mathbf{n}) \\ &= \frac{\partial}{\partial n}(\phi_t) + [(\mathbf{U} + \mathbf{\Omega}\times\mathbf{r})\cdot\frac{\partial\nabla\phi}{\partial n} + \nabla\phi\cdot(\mathbf{\Omega}\times\mathbf{n}) \\ &= \frac{\partial}{\partial n}(\phi_t) + \mathbf{U}\cdot\frac{\partial\nabla\phi}{\partial n} + \mathbf{\Omega}\cdot\frac{\partial}{\partial n}(\mathbf{r}\times\nabla\phi).\end{aligned} \tag{A5}$$

Furthermore, when the point is fixed on the body surface, equation (A1) can be substituted into the left-hand side of (A5). This gives

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial n}\right) &= \frac{\mathrm{d}}{\mathrm{d}t}[(\mathbf{U} + \mathbf{\Omega}\times\mathbf{r})\cdot\mathbf{n}] \\ &= [\dot{\mathbf{U}} + \dot{\mathbf{\Omega}}\times\mathbf{r} + \mathbf{\Omega}\times(\mathbf{\Omega}\times r)]\cdot\mathbf{n} + (\mathbf{U} + \mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{n}) \\ &= [\dot{\mathbf{U}} + \dot{\mathbf{\Omega}}\times\mathbf{r}]\cdot\mathbf{n} + \mathbf{\Omega}\cdot(\mathbf{n}\times\mathbf{U}).\end{aligned} \tag{A6}$$

Substituting equation (A6) into equation (A5), we have

$$\frac{\partial}{\partial n}(\phi_t) = [\dot{\mathbf{U}} + \dot{\mathbf{\Omega}} \times \mathbf{r}] \cdot \mathbf{n} - \mathbf{U} \cdot \frac{\partial \nabla \phi}{\partial n} + \mathbf{\Omega} \cdot \frac{\partial}{\partial n}[\mathbf{r} \times (\mathbf{U} - \nabla \phi)]. \tag{A7}$$